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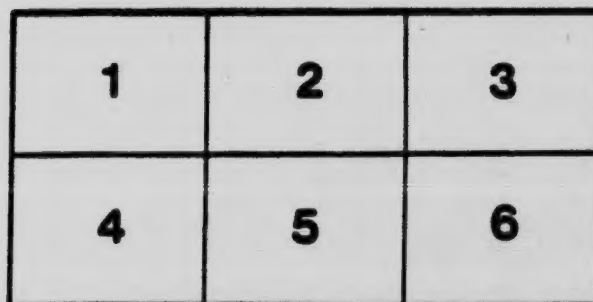
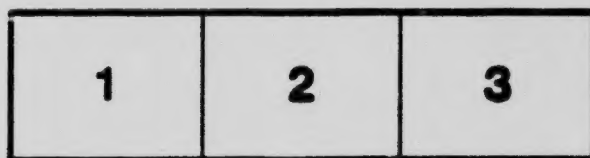
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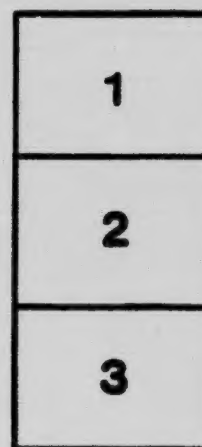
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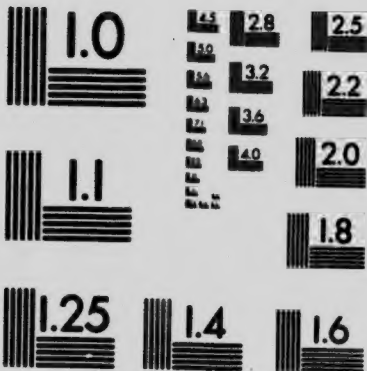
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ERRORS IN THE MEASUREMENT OF SPECTRAL
LINE DISPLACEMENTS

BY

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SEP 20 1912

ERRORS IN THE MEASUREMENT OF SPECTRAL LINE DISPLACEMENTS

BY RALPH E. DELURY

THE presence of systematic errors of a personal nature in the measurements of the positions of spectrum lines has often been suspected, yet little attempt has been made to determine their character and magnitude. In the present paper a method for investigating and eliminating these errors is described along with some measurements made in connection with the problem of the solar rotation, after first briefly mentioning some of the more important questions likely to be affected by such errors.

Systematic personal errors are suspected to occur in the measurements of the following investigations:

(a) *The determination of wave-lengths.* The greater part of this work involves repeated "settings" on the lines whose wave-lengths are sought and on other lines called "standards" whose wave-lengths are known. These "settings" consist in turnings of a micrometer-screw carrying the measuring microscope or the photograph of the spectrum, until the line or lines of the microscope are placed as closely as the measurer can see, over or symmetrically about the centres, centres of intensity or some other selected part of the spectrum lines and recording the corresponding scale-readings. It is obvious that if one observer uses the centre of the line for his settings and another uses the centres of intensity, systematic errors will occur, since for some lines the two "centres" coincide, while on others they do not. Again, even if two persons strive to set on the same part of the line such things as unsymmetrical shading of the line, the nearness of other lines, etc., may affect them differently and result in different settings with consequent errors in the determination of the wave-length. Such errors are so small, when the spectrum

is on a large scale, that for ordinary purposes the measurements are sufficiently accurate. In the determination of standard wavelengths, however, an effort should be made to eliminate even these errors. It is generally assumed that the mean of the measurements of several measurers is well within the limits of the accidental errors. This is not necessarily the case, however, for disturbing influences such as those mentioned above may operate on the measurements systematically in the same way for the different persons. There is apparently some evidence of this in the determination by interferometer methods of the "Secondary Standards of Wave-length, International System, in the Arc Spectrum of Iron,"* by Fabry and Buisson, Eversheim and Pfund, as the following quotation from H. Kayser's paper on "Standards of Third Order of Wave-lengths on the International System," † would seem to show: "I have employed as standards the arithmetical means of the measurements of Fabry and Buisson, Eversheim and Pfund. The portions of spectrum measured were always between three or four successive standards, so that each line is referred to as many neighboring standard lines as possible. It turned out that measurements of the same sharp line on different plates yielded differences of not more than from 0.001 to 0.002 Å when the same standards were employed; but if different standards were employed, differences as great as about 0.006 occurred. This proves that some of the standards contain errors of from 0.004 to 0.005 Å, and that measurements of the best plates in the second order give a greater degree of accuracy than that of the secondary standards." I cannot say that I am wholly convinced by the argument. It seems to me quite possible that Kayser's own settings on the secondary standards may contain systematic errors greater than the differences in the "measurements of the same sharp line on different plates." However, the fact remains that in the determinations of either the secondary or the tertiary standards, or both, *there are systematic errors greater than the accidental errors of measurement.*

(b) *The determination of stellar radial velocities.* In this

* *Astrophysical Journal* XXXII, 215-216.

† *Ibid.* XXXII, 217-225.

work systematic errors of measurement are more likely to occur than in any other involving the measurement of spectra, because the spectra measured—the star spectrum with its usually diffuse absorption lines, and the comparison spectrum with its strong emission lines—are so different in character. Furthermore, since the lines of the star spectrum often differ greatly in appearance and are in regions of the continuous spectrum of different intensity, systematic differences in velocity for the various lines are likely to be found, and since the star spectra are on a very small scale, these may represent variations of several kilometres per second. Such differences have been found, and though there are plausible physical explanations which one can give, such as electrical or magnetic effects, pressure effects, the presence of other bodies or of a hazy envelope about the star having a different motion, etc., yet it would be well before employing such theories to eliminate the possibility of systematic errors of measurement.

(c) *The comparison of various regions in the sun with one another and with standard spectra.* Much of this work depends on comparative measurements of spectra of different character, and, as pointed out above, is therefore liable to contain systematic error. Among the more important investigations in this field are some of those carried out at Mount Wilson Solar Observatory, on the displacements of the spectrum lines at the sun's limb*, and the motions of calcium vapor in the solar atmosphere† and over special regions in the sun.‡ The accidental errors of measurement in this work are extremely low, a precision of $0.0001\text{\AA}$ being aimed at, and such being the case it would be well worth while to test for systematic errors which may well exceed this small quantity. Furthermore, in deciding whether the interesting results observed in these researches are due to pressure or to convection very accurate references to laboratory comparison spectra were necessary, and the spectra compared being widely different the decisions may have been affected by systematic error. Even

*Walter S. Adams, *Astrophysical Journal*, XXXI., 30-61.

†Charles E. St. John, *Ibid*, XXXII, 36-82.

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so may be affected the results derived from the comparison of "enhanced" and "arc" lines in the same spectrum.*

(d) *The determination of the rate of the solar rotation.* The method chiefly used at present in the investigation of the rate of the sun's rotation, is that employed by Adams† at the Mount Wilson Solar Observatory, namely, the measurement of the spectral line displacements in adjacent simultaneously taken photographs of the spectra from opposite ends of a diameter of the sun's disc. These measurements differ from those made in the investigations mentioned above, in that the relative positions of *the same spectral lines in two presumably similar spectra* are sought, and consequently it is not so easily imagined, as in the case where *two different spectra* are compared, how systematic errors of measurement may happen. Nevertheless, it was in connection with this very problem that I first suspected the presence of such error. In 1909 I measured a few plates of the solar rotation effect which I made with the twenty-three foot Littrow spectrograph using a plane grating which apparently on account of curvature of the lines‡ produced spectra of very poor definition. The spectrum lines were not sharp and the continuous spectrum was nebulous. From my measurements of from 30 to 80 lines on each of three plates, I noted that the greater proportion of the lines giving the largest values of the measured displacements were the broader and more fuzzy lines or those very close to other lines or to nebulous regions of the spectrum. A few scattered measurements on other plates seemed to confirm this observation. Measurements of one of these plates were published along with a description of the Spectrograph§ simply to illustrate the method, but even from these 80 measurements the tendency for the broader and more diffuse lines to give larger values for the displacement is noticeable. The mean velocity-equivalent of the measured displacements is 1.77 km. per second, and of the 9 lines with

*Walter S. Adams, "Some results of a study of the spectra of Sirius, Procyon and Arcturus with high dispersion," *Astrophysical Journal*, XXXIII., 64-71.

†*Ibid.*, XXVI., 203; XXIX., 110; XXVII., 213.

‡L. Lury, *JOURNAL ROYAL ASTRONOMICAL SOCIETY OF CANADA*, V., 26-32.

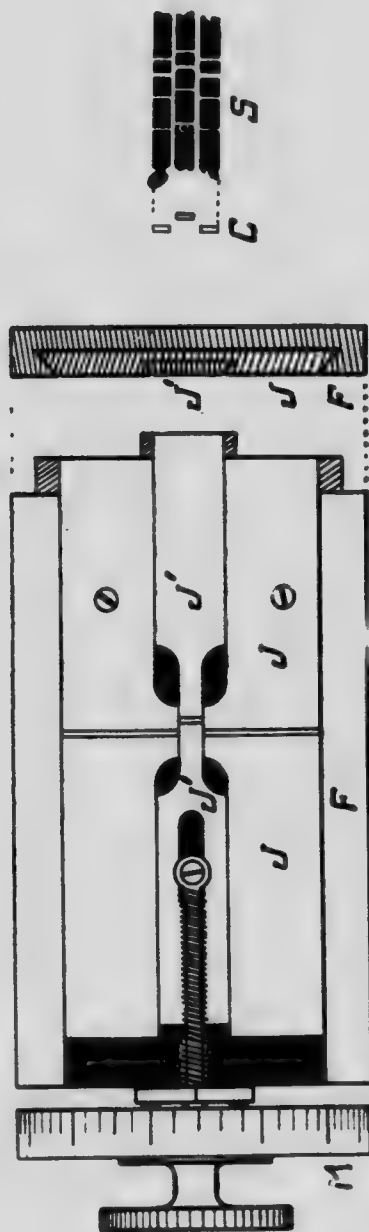
§*Report of the Chief Astronomer*, 1909, 255.

values greater than 1.90 km. per second, 4 are exceptionally broad or fuzzy lines, 1 is in a nebulous region and another is very close to another spectrum line, as follows ; 4163.818, fuzzy, 1.95 km. ; 4199.257, fuzzy, 2.13, and 1.95 km. ; 4236.279, near 4236.112, 1.92, and 1.94 km. ; 4246.966, broad and fuzzy, 1.94 km. ; 4271.325, broad and fuzzy, 1.91 km. ; (4271.934, broad and fuzzy neighboring line gives a value nearly as great, 1.87 km.) ; 4288.310, nebulous region, 1.98 ; the other three lines appear to be average lines. These measurements with the others mentioned above certainly warranted the suspicion that the broader and more nebulous lines gave larger values, and so much was I impressed by them that my suspicions extended to the measurements of Adams on the rotation (*loc. cit.*) values for the two very broad and shaded lines H_{α} and λ 4227, which he found to give larger values for the rate of the sun's rotation than those obtained from the narrower absorption lines. I felt the necessity of making measurements to test these suspicions but thought it better to wait until a new grating giving sharper spectrum lines was obtained. Consequently, when work with the new grating was well under way and some measurements were made which still showed systematic tendencies, particularly some comparative measures by different persons, I proposed to make tests for systematic errors of measurement to be carried out in connection with the measurements of the plates of the solar rotation effect, in the hope that these errors if they actually existed could be eliminated and also that light might be thrown on my suspicions as to the reality of the results obtained by Adams regarding the systematically different displacements of various lines. I accordingly devised a method of attacking the problem, a description of which, together with some preliminary measures, constitutes the the remainder of this paper.

The method consists essentially in introducing by mechanical means on the spectrum lines under investigation an arbitrary displacement which is the same for all the lines, and which should yield, in the absence of systematic errors, the same measured values within the limits of the accidental errors, and for a large

series of observations the same means for all the lines whether measured by the same or different observers; if the means of the measured values in such a series are not close to the same value, then systematic errors are present, and their magnitude and nature can be determined from the differences.

The arbitrary displacement may be introduced by taking the photograph of one spectrum first, then shifting the plate-holder and taking the photograph of the other spectrum. The plate-holder may be moved between stops or moved by means of a micrometer screw. The method is very satisfactory if one is not anxious to have the displacements on a series of plates identically the same. For several reasons, however, I desired the latter condition and accordingly devised a second method of obtaining the displacement. This method consists in the taking of spectra from two parallel slits whose widths and distances apart are adjustable and may be kept constant for a series of plates, the distance apart of the slits governing the displacement of the spectra. To reproduce the configuration of the "rotation spectra" taken here, I had the "double-slit" constructed in the form shown in the Figure, which represents a bird's eye view of the apparatus. *F*, is a brass frame with bevelled runs in which the slit-jaws, *J*, *J'*, work. These jaws are also milled with bevelled runs in which the smaller jaws *J''*, *J'''*, slide. The slit edges of all these jaws are parallel, and they are bevelled back, leaving their sharp edges in the plane of their faces. The smaller jaws were set in the larger and their edges polished simultaneously to insure parallelism, the polishing being done on plate-glass with the finest emery. One of the jaws, *J*, is screwed to *F* in a fixed position, while the other may be slid back and forth in *F* by means of the micrometer, *M*, with the aid of a coiled spring as in ordinary spectograph-slits, and the distance between the jaws may be read in ten-thousandths of an inch. The slit between the jaws, *J''*, *J'''*, may be adjusted to any desired distance to either side of the slit between the larger jaws. The widths of the two slits will be the same if they are adjusted so that they close simultaneously, since when jaw *J* is moved by the micrometer it carries with it one of the smaller



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jaws. A mask is next placed over the slits to give the configuration *C*. The double-slit replaces the ordinary slit of the spectrograph, and the spectra taken are represented by *S*. A precisely similar mask is used in taking the rotation spectra, so that the widths of the strips of spectra and the distances apart are exactly the same. Both spectra are taken from the same distance inside the limb of the sun, so that the only difference between them lies in the fact that in the rotation spectra the displacement of the spectral lines is due to the rotation of the sun, while in the double-slit spectra the displacement is caused by the displacement of the slits and is, therefore, the same for all lines. Measurements of the latter should therefore reveal the exact nature of the errors of measurement of the former. In a somewhat similar manner the double-slit may be employed to investigate the errors of measurement in connection with the other problems mentioned above. If the double-slit is to be used very much it will be found advantageous to provide a micrometer attachment to the right-hand jaw, *J'*, so that its distance from the slit made by the larger jaws may be readily adjusted. For use in the present investigation the displacement was fixed at about 0.07 mm., the value of the displacement at the equator on the rotation plates at λ 4250, the region chosen because it was that employed by Adams in his work on the Solar Rotation and is the common region selected by the International Union for Co-operation in Solar Research.

In making the measurements the large Toepfer measuring machine of the Observatory was used. This instrument has a thread 30 cm. (12 in.) long, and of 0.5 mm. pitch and is capable of measuring over the whole range of the 12 in. plates taken in the Solar Spectrograph. The micrometer reads to microns, and estimations to one-tenth of this value, *i. e.*, to one ten-thousandth of a mm., (0.0001 mm.) may be made. The measures given in the following Tables are expressed in this unit. All measurements of the displacement of any line consist in taking the difference between the means of 4 settings on the line in the central strip of spectrum and the mean of 2 settings on the line in each of the two outside strips; when all the lines in question are

measured, the plate is turned through 180° (always face up) and the measurements repeated, the run of the micrometer screw being in the opposite direction: the mean of the means from the two runs d' and d'' , for each line is the measured displacement, d , for that line, and it thus depends on 16 settings. Before giving the measurements of the arbitrary displacement plates, I will present some results illustrating the differences in the measures of the same lines by different observers. In the Tables, d denotes displacement, and r the residual of the displacement obtained by subtracting the mean from each value; $p. r.$ denotes the probable residual (for the r 's in question).

TABLE I.—PLATE, L570a, (ROTATION EFFECT).

Line Measurer	4548.938		4558.827		4564.939		4571.275		Means	
	d	r	d	r	d	r	d	r	d	r
Plaskett	533	- 4	537	+ 14	540	+ 25	545	+ 21	539	+ 14
	(571	+ 34)	(553	+ 30)	(566	+ 51)	(577	+ 53)	(568)	
Harper	563	+ 26	515	- 8	510	- 5	510	+ 26	535	+ 10
Parker	550	+ 13	540	+ 17	500	- 15	520	- 4	528	+ 3
Cannon	540	+ 3	552	+ 29	513	- 2	502	- 22	527	+ 2
Motherwell	543	+ 6	500	- 23	500	- 15	522	- 2	516	- 9
De Lury	493	44	496	- 27	526	+ 11	507	- 17	506	19
	(520	- 17)	(490	- 33)	(481	- 34)	(536	- 12)	(507)	
Means	537		523		515		524		525	

Note.—The bracketed values are duplicate measures previously made and whose peculiar differences made it desirable to have measurements made by other observers. All but the bracketed values are derived from measures made in only one position of the plate.

TABLE II.—L701f, ARBITRARY DISPLACEMENT BY DOUBLE-SLIT.

Line	4220.509, Fe, 3	4225.619, Fe, 3	4232.887, Fe, 2	4241.285, Fe, 2	4246.996, Fe, 2, 5	Means
Measurer	d'' d''' d	d'' d''' d	d'' d''' d	d'' d''' d	d'' d''' d	d'' d''' d
Plaskett	721 704 713 (712 697 709)	720 730 725 (687 710 699)	700 713 706 (702 687 695)	707 700 704 (715 718 716)	689 683 686 (688 690 689)	707 706 707 (701 700 700)
Cannon	706 701 704	679 725 702	678 660 669	710 668 689	687 731 709	692 697 694
Parker	648 716 682	663 744 704	709 671 690	691 725 704	663 688 675	675 709 692
Harper	676 700 688	700 742 724	654 666 660	714 681 697	669 671 670	684 692 688
De Lury	669 682 676 (669 731 700)	733 701 717 (741 726 734)	649 659 654 (651 638 664)	710 702 700 (648 652 650)	653 690 671 (715 660 688)	683 687 685 (693 681 687)
Stewart	658 687 673	656 712 681	649 666	663 669 666	705 658 682	673 675 674
Means	d'' 685 d''' 698 d 689	693 726 709	679 669 674	639 689 694	678 686 682	686 694 690

Note.—The bracketed values, previously obtained, suggested the advisability of repeating the measurements and having other observers make them also. The bracketed values are not included in taking the means. The values obtained for the whole exposure L701f at the previous measurement, were, Plaskett, 702; De Lury, 680.

TABLE IV.—L 699, ARBITRARY DISPLACEMENT, BY DOUBLE-SLIT.
Measured by DeLury

Line	L699a			L699b			L699c			L699d			L699e			L699f			Means				
	d'	d''	d	d'	d''	d	d'	d''	d	d'	d''	d	d'	d''	d	d'	d''	d	d'	d''	d		
1, 4196.699, La,	2	654	639	647	646	730	688	736	795	764	715	721	718	663	719	691	721	689	708	693	716	703	+ 10
2, 4197.257, CN,	2	614	655	634	708	797	798	711	704	708	675	719	697	633	780	707	690	674	682	687	721	704	+ 11
3, 4216.136, CN,	2	723	679	701	678	695	686	727	678	702	641	748	695	713	730	721	743	740	741	704	712	708	+ 17
4, 4220.509, Fe,	3	626	638	632	724	725	725	710	777	744	729	681	705	705	670	688	654	715	685	691	701	696	+ 4
5, 4225.619, Fe,	3	726	743	735	634	631	632	793	711	752	641	638	639	667	692	679	742	712	727	700	688	694	+ 1
6, 4232.887, Fe,	2	696	668	682	697	687	692	755	731	743	675	675	675	666	689	678	611	660	635	683	685	684	- 9
7, 4241.285, Fe/Zr,	2	711	666	688	671	646	658	611	655	633	685	781	733	720	735	727	850	767	808	708	708	708	+ 15
8, 4246.996, Sc,	5	642	637	639	660	58	644	693	727	710	604	747	676	689	729	709	716	792	754	607	711	689	+ 4
9, 4257.815, Mn,	2	642	743	693	604	595	599	600	701	650	657	713	685	631	722	677	705	767	730	640	707	673	- 20
10, 4258.477, Fe,	2	687	698	692	673	673	673	699	705	702	712	653	683	705	672	688	664	733	698	690	689	689	- 4
11, 4266.081, Mn,	2	641	635	638	606	738	717	626	726	676	722	686	704	682	670	676	650	689	673	671	691	681	- 12
12, 4268.915, Fe,	2	590	589	589	719	659	689	690	603	644	708	681	694	650	696	673	704	686	695	677	653	665	- 23
13, 4276.836, Zr,	2	713	717	715	660	680	670	610	658	634	724	724	724	698	705	702	710	675	692	686	692	689	+ 4
14, 4290.377, Ti,	1	716	771	744	764	747	755	697	719	708	730	663	697	700	682	691	640	715	678	708	716	712	+ 19
15, 4291.630, Fe,	2	733	761	747	670	657	664	670	663	667	740	683	711	710	692	700	668	709	688	700	692	696	+ 3
Means d', d'', d	674	683	678	686	686	686	686	689	704	696	691	701	696	683	706	694	699	715	707	687	689	693	10.7

Measured by DeLury

Line	I-701a			I-701b			I-701c			I-701d			I-701e			I-701f			Means				
	d'	d''	d	d'	d''	d	d'	d''	d	d'	d''	d	d'	d''	d	d'	d''	d	d'	d''	d		
1, 4196·090, La.	2	641	606	624	673	736	704	663	716	640	700	742	721	675	724	699	678	727	717	671	697	684	10
2, 4197·257, CN.	2	647	586	616	654	748	701	622	610	616	758	785	772	674	675	675	586	620	603	657	671	664	30
3, 4216·136, CN.	2	651	740	696	636	675	686	728	713	720	672	692	682	696	732	714	703	726	715	685	719	702	+ 8
4, 4220·509, Fe.	3	702	728	745	748	704	726	707	673	690	642	667	654	687	687	687	669	731	700	702	698	700	+ 6
5, 4225·619, Fe.	3	686	660	673	754	811	782	729	744	737	791	655	723	747	700	723	741	726	733	740	716	728	+ 34
6, 4232·887, Fe.	2	707	651	679	714	736	725	699	702	700	724	713	719	706	670	688	691	638	665	713	685	699	+ 5
7, 4241·285, Fe-Zr.	2	718	755	736	620	681	651	693	737	715	756	747	751	656	672	664	648	652	650	683	707	695	+ 1
8, 4246·996, Sc.	5	660	647	654	614	671	656	674	717	704	636	671	704	645	681	664	715	660	687	639	701	665	29
9, 4257·815, Mn.	2	675	705	690	765	762	763	744	699	722	623	660	641	685	719	702	816	727	772	718	712	715	+ 21
10, 4258·477, Fe.	2	697	709	703	694	651	673	698	714	706	666	683	675	682	695	683	680	696	683	687	691	689	- 5
11, 4266·081, Mn.	2	673	677	675	618	614	616	753	765	759	721	748	724	695	636	666	641	650	645	680	682	681	- 13
12, 4268·915, Fe.	2	655	663	659	605	741	703	630	662	649	682	652	667	700	701	700	633	629	631	661	675	668	- 26
13, 4276·836, Zr.	2	746	749	747	697	703	700	737	725	730	709	665	687	668	656	660	667	709	686	703	701	702	- 8
14, 4290·377, Ti.	1	752	733	743	733	742	737	716	711	714	736	735	735	734	727	730	661	686	678	724	722	723	+ 29
15, 4291·630, Fe.	2	724	710	717	636	686	661	727	676	701	659	612	630	711	672	692	710	738	724	694	682	688	- 6
Means d'	693	687	690	687	711	699	702	702	691	686	697	701	699	683	689	684	690	697	694	± 15·4			
Means d''																							
Means d							699	702	702	690	697	701	699	686	686	686	690	697	694				

TABLE VI.
ERRORS OF SETTING.

L699, ARBITRARY DISPLACEMENT BY DOUBLE-SLIT.

Line	Algebraic Means					Arithmetic Means				Means
	1st	2nd	3rd	4th	1st	2nd	3rd	4th		
1, 4196.699, La,	2	- 7	+ 13	- 6	- 1	18	26	12	18	18.6
2, 4197.257, CN,	2	- 0	+ 7	0	- 11	28	25	25	20	24.5
3, 4216.136, CN,	2	+ 7	- 1	+ 8	- 3	40	23	30	31	31.0
4, 4220.509, Fe,	3	+ 9	+ 2	- 2	- 3	24	16	22	23	21.3
5, 4225.619, Fe,	3	+ 20	- 12	- 8	0	25	20	28	20	23.3
6, 4232.887, Fe,	2	- 15	+ 12	- 4	+ 6	21	17	20	14	18.0
7, 4241.285, Fe-Zr,	2	0	- 7	+ 6	0	25	12	18	21	18.8
8, 4246.996, Sc,	5?	14	+ 5	- 7	+ 18	25	17	21	19	20.5
9, 4257.815, Mn,	2	+ 10	- 4	- 6	0	34	19	13	24	22.5
10, 4258.477, Fe,	2	+ 3	- 2	- 2	+ 1	21	13	12	15	15.3
11, 4266.081, Mn,	2	14	+ 1	+ 12	+ 2	21	10	10	14	17.5
12, 4268.915, Fe,	2	- 3	- 14	+ 4	+ 12	21	22	11	21	18.8
13, 4276.836, Zr,	2	4	- 7	- 6	+ 16	14	13	16	20	15.8
14, 4290.377, Ti,	1	+ 3	0	- 2	- 2	14	18	14	13	14.8
15, 4291.630, Fe,	2	2	- 8	+ 8	+ 3	14	14	18	10	14.0
Means	- 1	- 1	0	+ 2		23	18	18.6	19	19.6

Note.—The above numbers are the means—algebraic (with regard to sign), and arithmetic (without regard for sign)—of the 1st, 2nd, 3rd and 4th settings on the lines in the central strips of the six exposures, L699a to L699f, measured both ways; each mean for each line is therefore the mean of 12 settings made at different times.

TABLE VIII—SUMMARY OF MEASUREMENTS OF ARBITRARY DISPLACEMENTS, PLATES L689 AND L701.
Measured by Plaskett.

Line	Int.	Width	Remarks	(a)	(b)	(c)	(d)	(e)	(f)	(a)	(b)	(c)	(d)	(e)	(f)	Means
1. 4100-500, L.A. 2,		2020	Diffuse	683	692	710	704	709	700	693	717	683	693	711	704	700
			Strong	0	2	8	13	5	3	15	11	23	1	14	2	0 - 7.6
2. 4197-257, CN, 2,		1660	Asymmetrical	706	716	714	705	719	700	717	702	715	696	681	690	700
			Weak	+14	+22	+12	+14	+15	3	0	4	9	2	10	3	6 8.4
3. 4216-136, CN, 2,		2052	Asymmetrical	688	710	689	690	706	702	704	709	689	686	703	705	700
				-4	+16	-13	+5	+2	1	4	3	7	5	6	3	0 5.2
4. 4220-509, Fe, 3,		1710	Good, strong	686	693	705	688	695	704	710	694	692	696	706	709	698
			Narrow	-6	0	3	3	0	1	3	12	14	2	9	7	2 5.0
5. 4225-619, Fe, "		1990	Good, strong	703	688	700	686	705	704	696	703	710	695	683	699	698
			Near 4227	+11	-0	2	5	+1	1	12	+2	4	-2	14	3	2 4.8
6. 4232-887, Fe, 2,		1634	Fine, good	689	677	695	704	684	696	715	707	703	689	705	697	-3 6.8
				3	17	-7	+13	-20	7	-7	+1	-3	-5	+8	7	-3 6.8
7. 4241-285, Fe-1/2, 2,		1409	Fine, weak	692	708	704	696	691	710	710	711	707	702	689	716	703
				0	+14	+2	+5	-13	+7	+2	+5	+1	+8	-8	+14	+3 5.8
8. 4246-006, Sc, 5,		2500	Strong, broad	680	675	693	682	695	700	707	699	701	685	687	680	692
				3	19	9	9	9	3	-1	-7	5	9	10	13	-8 6.4
9. 4257-815, Mn, 2,		1640	Diffuse	702	675	693	675	717	708	713	711	713	700	698	698	700
			Weak	+10	-19	-9	-16	+13	+5	+5	+5	+7	+6	+1	4	0 6.7
10. 4258-477, Fe, 2,		1025	Broad	691	704	697	691	691	703	715	705	710	689	685	701	699
			Strong	1	+10	5	0	13	0	7	1	10	5	12	1	1 5.1
11. 4266-081, Mn, 2,		1710	Fair	679	706	721	682	707	712	698	706	721	700	687	689	701
			Weak	-13	+12	+19	9	+3	+9	-10	0	+15	+6	-10	13	+1 7.3
12. 4268-915, Fe, 2,		1480	Good	681	704	698	682	717	707	712	711	708	696	696	710	702
			Edges weak	-11	+10	4	9	+13	+4	-4	5	+2	2	-1	-8	+2 5.1
13. 4270-830, Zn, 2,		1500	Diffuse	691	691	708	684	700	686	711	705	698	702	712	704	700
				-1	3	6	7	4	17	+3	1	-8	-8	+15	7	0 5.8
14. 4290-377, Ti, 1,		2365	Broad	699	686	705	690	700	696	703	695	713	693	709	704	700
			Edges sharp	-7	-8	+3	8	+2	7	5	11	7	1	12	2	0 4.8
15. 4291-630, Fe, 2,		1900	Broad	700	681	695	686	715	710	713	709	711	684	705	700	701
				+8	13	7	5	+11	+7	+5	+3	+5	-10	+9	2	+1 5.8
Mean				692	693	702	691	704	703	708	706	700	694	697	702	700
Probable residual				9.1	6.0	6.4	7.2	4.0	5.0	4.2	6.8	3.9	7.4	5.0	5.6	Mean $\bar{p} = 5.9$

Probable residual of the displacement for a single exposure = 4.1.

The discussion of the results given above can best be made under the headings of the various classes of errors investigated:

Systematic differences in the measurements by different measurers.

A glance at Tables I. and II. reveals the fact there are systematic differences in the measurements made by different persons, and in the general trend the results indicate that those who have had considerable experience in the measuring of stellar radial velocity spectrograms get higher values than those who have had little experience in the measurement of spectra. The differences between the highest and lowest values of the mean displacements are, from Table I., 33, *i. e.*, 0.0033 mm., and from Table II., the same value, 33, representing differences of 6.3 per cent, and 4.8 per cent respectively. These differences are equivalent to differences of 0.13 and 0.10 km. per second in the determination of the equatorial rate of the sun's rotation from similar displacements. This is a striking difference, and the question arises: Which is more nearly the true value, the high measurement, or the low one? and further, what is the true value?—for obviously it is not safe to take the mean value of all the measurements when such systematic errors occur. From the results of a large series of measurements, involving double measures on 12 different exposures (summarized in Tables VII. and VIII.) Mr. Plaskett's measurements are seen in the mean to be greater than mine by 6 in a displacement of about 700. This difference is systematically in the same direction with the exception of the measurements for 2 out of the 12 exposures, and 3 out of the 15 lines, one line giving the same mean. This difference corresponds to 0.02 km. per second, a very small quantity, yet it is in the hundredths of a kilometer per second that nearly all of the interest in the determination of the solar rotation lies at the present time in view of the many interesting results obtained by Adams and Halm. It is consequently of considerable importance in such investigations to make allowance and correction for the systematic differences in the measurements of different persons.

Systematic differences depending on the configuration measured.

A very curious systematic difference in the measurement of the

displacements presents itself when measures are made one way and the plate is reversed and measured in the other direction. The face of the plate being up in all measurements. This d'' which apparently is in the same sense for all observers, is exhibited in Table II. where measures of 6 lines on one exposure are given for six different measurers, and in Tables III., IV. and V. where my own measurements of 16 different exposures are recorded. It will be seen from Table II. that, with the exception of Mr. Plaskett's measurements which in this Table do not show the effect, the first measurement, d' , is systematically smaller than d'' , the second measurement, for each line with one exception, and for each observer. In the mean, $d'' - d' = 694 - 686 = 8$, a difference corresponding to 0.023 km. per second. In my larger series of measurements this peculiarity is even more strikingly shown. In the values for L641, Table III., we have, $d'' - d' = 820 - 806 = 14$, which corresponds in the region of the spectrum measured, to about 0.04 km. per second. For L699, Table IV., the difference, $d'' - d' = 699 - 687 = 12$, corresponds to 0.03 km. per second; and for L701, Table V., the difference, $d'' - d' = 697 - 690 = 7$, corresponds to 0.02 km. per second. In all cases for the measurements of d' the configuration of the lines as seen in the eye-piece of the measuring machine, was,  while for the measurements d'' it was the opposite, . Furthermore, the carriage was advanced so that the spider-line of the eye-piece always passed over the central line first, and the four settings were made on the central line before passing to the two outside lines. It is not likely that the fact that the d'' measures were made subsequently to the d' measurements has anything to do with this remarkable difference; (yet there is the possibility of one's measurements becoming more "liberal" as he continues to measure — a "fatigue" effect) nor are temperature differences in the micrometer screw caused by the presence of the measurer while securing the values of d' sufficient to account for the larger values of d'' . Nor does the direction of turning the screw account for the difference,

for in the measurements of L641 the screw was turned in the opposite direction from that used in the other measurements, the central strip being displaced to the red of the two outside strips in it and to the violet side of the spectrum in plates L699 and L701: to get the same configuration in the microscope for the d' measures it was therefore necessary to start the readings from the opposite ends of the scale for the two opposite displacements. It thus appears that the difference does not depend on the direction of turning of the screw but on the configuration of the three lines visible in the eye-piece of the measuring machine. Thus, the most plausible explanation of the difference, it seems to me, is a "right-handed and left-handed" effect. The fact that in d' the lines in the two outside strips appear to the right of the line in the central strip, and that in d'' the opposite is the case may produce effects on the eye resulting in this curious difference in the measures of the displacement. I try to eliminate such a thing as "muscular memory" by taking a fresh grip on the turning wheel before each setting. Further-more it must be understood that the magnification of the microscope is so great that when a setting is made on one of three lines, the observer is *apparently* not conscious of the presence of the other lines; the above considerations would seem to indicate that the other lines exert some small influence over him nevertheless. In Adams' work on the rotation two strips of spectra were employed, so that the configuration of the lines would be the same in both positions of the plate, and consequently the measured displacements are liable to be too large or too small from such an effect as is here shown to be present. This effect is so large that it must be eliminated from the measurements.

Systematic differences for different lines.

From Table VII. it will be seen that there is a tendency towards negative residuals in my measurements in the cases of lines 8, 11 and 12; and positive residuals for the lines 5 and 14. In Mr. Plaskett's results the tendency to negative residuals occurs in the cases of lines 6 and 8, and positive residuals for lines 2 and 7. Nearly all of these lines have some exceptional characteristic:—

line 2, 4197.257, CN, is a weak line slightly diffuse and shaded asymmetrically; line 5, 4225.619, Fe, is a good strong line with slightly diffuse edges and it is near the strong, broad and nebulous line of Ca, 4227, which appears in the field of the microscope during the measurements of line 5; line 6, 4232.887, Fe, is a narrow strong line and a good one for measuring; line 7, 4241.285, Fe-Zr, is narrow and weak; line 8, 4246.996, Sc, is a strong and very broad line but it has sharp edges and is therefore a fairly good line to measure; 11, 4266.915, Mn, 2, is a weak line; line 12, (4268.915, Fe, is a narrow line with edges slightly diffuse; line 14 4290.377, Ti, is a very broad line with its edges sharp and strong. As seen in the last paragraph it seems very probable that the configuration of the line plays some part or has some slight effect on the measurements; such being the case it is reasonable to suppose that this effect will vary with lines of different character, and consequently the measured displacements may vary. I made measurements on three dense exposures of the Ca line 4227 to see if any exceptional effects were present such as those obtained for this line and for Ha by Adams in his solar rotation work. The errors of setting were very large and nothing definite could be found from so few measures. Such measures should be made, and preferably under the same conditions as the rotation plates were measured by Adams and Miss Lasby. The present results show, however, that there is a danger of systematic errors being present for the various lines, and the differences obtained for individual lines in the rotation work should be very carefully examined by the person making the measurements, at the time they are made preferably, to see if they are due to such systematic errors of measurement.

It will be seen from Tables VII. and VIII., that the probable residuals for the different lines differ greatly, and that my *p.v.* is much greater than Mr. Plaskett's. This is probably due to the fact that the latter observer has had a great deal of experience in the measurement of the broad diffuse lines of stars spectra, and his eye is accustomed to smoothing out the irregularities in the lines due to the grain of the plate, and that I am still bothered

by it and have more difficulty in setting on the centres of intensity of the lines. It is likely that more experience will relieve me of much of this error. These errors are combined accidental and systematic, and probably when the systematic errors are removed the residuals will be greatly lessened.

Systematic errors in setting.

In Table VI. are given my errors of setting for the 4 settings on the lines in the central strips (measured both ways) of Plate, L699. The numbers are the means of the residuals, with regard to sign and without regard to sign, obtained by subtracting the mean of the 4 settings on any one line from each of the settings on that particular line and taking the means for each line. Though each line has its own peculiar arrangement of the positive and negative residuals for each of the 4 settings—quite striking in some cases—yet there seems to be no general systematic arrangement of these signs. The means without regard to sign show the peculiar and not unexpected result that the first settings are usually farthest from the mean, while the residuals for the other three settings are of about the same magnitude. This peculiarity may be lessened to some extent by viewing the lines as a whole more carefully before taking any settings. The mean error of setting is, ± 19.6 , or about 0.002 mm., equivalent to nearly 0.056 km. per sec.

The above results were obtained in measurements of the simplest class, namely, the relative positions of the *same* lines, and where, therefore, systematic errors are not so likely to occur as in investigations where the relative positions of *different* lines are determined; yet, nevertheless, the results show the presence of systematic errors. It may be taken for granted, then, that similar and larger errors occur in other spectroscopic measurements. And since the errors are sufficiently large that their removal is desirable, a general method for the elimination of such errors is necessary. An obvious method of eliminating some of these errors is to make the configuration the same for each measurement. This may be accomplished in the

particular case examined above by means of a moveable slit which could be readily placed so that only one strip of spectrum could be seen at a time, and each measured in turn. In the general case, however, where different lines are compared, this would not remove the individual errors of each line, and would not remove any systematic errors of "memory" if they were present. The "double-slit" used above offers a more complete elimination of the difficulty as follows:

By means of the double-slit introduce a displacement, D , on the spectra whose normal displacement, d , is desired to be found. By making D much larger than d , it may be assumed (and this point could easily be tested by use of the double-slit) that the systematic errors in measuring $D + d$ are the same as for the measurement of D . Thus let the measured value of the displacement be $D + d + e$, then the measured displacement of D will be $D + e$, and the difference between the two measurements, $(D + d + e) - (D + e)$, will give the true value, d , of the displacement sought. The principle is precisely the same as the "method of differences" used in exact weighings. It would involve two parallel series of measurements, but the determination of $D + e$ would probably not need to be made so frequently as the determination of $D + d + e$, after one became somewhat fixed in his habits of measurement. In the investigation of the solar rotation this method could be readily applied. The displacements, d , in our work here range from a little less than 0.1 mm. down to nearly zero, so that by introducing a displacement $D = 1$ mm. in all the rotation spectra the total displacements would be so nearly constant that very probably the systematic errors of measurement would be the same for all, and one determination of $D + e$ for spectra from 6 or 7 different latitudes on each plate would serve to eliminate the systematic errors in the mean. Absolute values, within the limits of the accidental errors, would thus be determined for d , and the results of the measures made by different observers and for different lines should agree very closely. I hope to test the method in the near future and eliminate the errors now present in our work.

Briefly the more important results of the present investigation are:

(1) A method is described for determining the presence and the nature of the errors in the measurements of spectral line displacements.

(2) There are systematic personal errors in the measurements of spectral line positions and displacements.

(3) The errors of measurement, both accidental and systematic, depend on the configuration measured, that is, on the grouping of the several lines in the field of the measuring microscope, and on the characters of these lines.

(4) The effect on the measurements of different observers, of the configuration employed in the present investigation is in the same sense.

(5) The systematic errors discovered in the present case are sufficiently large to introduce grave errors into one's results and into their interpretation, and consequently one's measurements cannot be accepted as sufficiently accurate without the elimination of these errors. Systematic errors ranging from 0.13 km. per sec. to 0.02 km. per sec. were found in measurements such as made for the determination of the rate of the solar rotation, the value for which at the equator is a little over 2 km. per sec.!

(6) Methods for the elimination of these systematic errors are described.

That the systematic errors which occurred in the measurements of the arbitrary displacements, actually occur in the measurements of rotation plates, is seen from the following comparative values of the equivalents (in kilometers per second) to the measured displacements on various plates:

Plates	L528a	L531a	L531b
Plaskett	1.866	1.868	1.872
De Lury	1.851	1.851	1.959

Plate 845a

Plaskett	1.712	1.682	
Cannon	1.632		
Parker	1.624		
Harper	1.618		
De Lury	1.592	1.618	1.624

The necessity for the removal of the errors in our work is thus emphasized. I hope to carry on further work on this question of systematic errors of measurement, and to investigate such influences as intensity of the photographic plate, the magnitude of the displacements, the character of the line with particular reference to the broad lines λ 4227, and H α ; and especially to test the methods proposed above for the elimination of these errors.

I wish to thank the gentlemen who so kindly made the comparative measurements given here; and Mr. Lucas for constructing the double-slit to work so efficiently.

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